When is Enough Good Enough in Source Modeling?

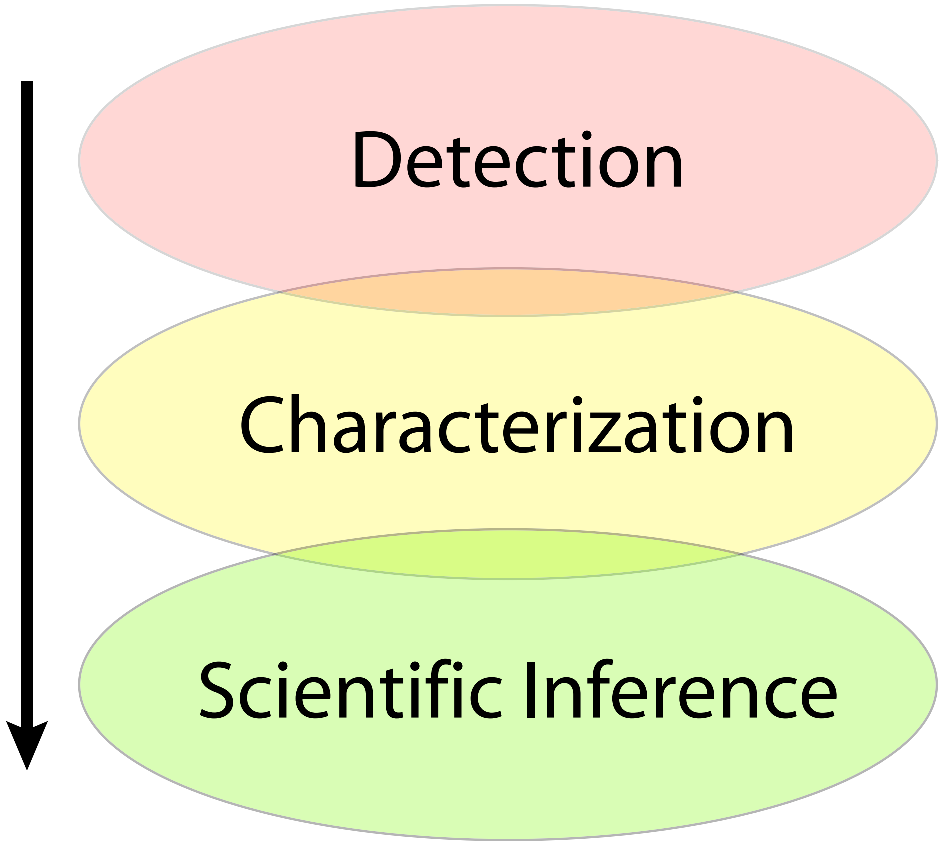
Louis J. Rubbo

rubbo@gravity.psu.edu

Center for Gravitational Wave Physics

The Pennsylvania State University

Data Analysis Flow Chart



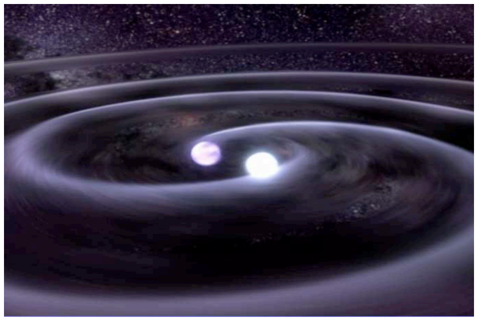
Is there a signal present in the data?

How is the signal parameterized and what are the estimates for the parameters?

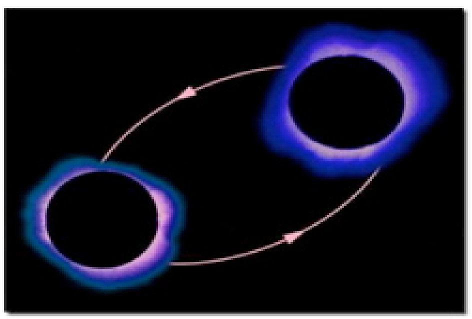
What new science have we gained from the data?

Talk Outline

1. Bayesian Model Comparison



2. Galactic Binary Evolutions



3. Other Applications

Bayes' Theorem

- $P(A|B) =$ probability of proposition A conditional on proposition B being true

- Bayes' Theorem:

$$P(\mathcal{H}_\alpha|\mathcal{D},\mathcal{I}) = P(\mathcal{H}_\alpha|\mathcal{I})\frac{P(\mathcal{D}|\mathcal{H}_\alpha,\mathcal{I})}{P(\mathcal{D}|\mathcal{I})}$$

$\mathcal{H}_\alpha \equiv$ Hypothesis $\quad \mathcal{D} \equiv$ Data $\quad \mathcal{I} \equiv$ Prior Information

Bayes' Theorem

- $P(A|B) =$ probability of proposition A conditional on proposition B being true
- Bayes' Theorem:

$$P(\mathcal{H}_\alpha|\mathcal{D},\mathcal{I}) = P(\mathcal{H}_\alpha|\mathcal{I})\frac{P(\mathcal{D}|\mathcal{H}_\alpha,\mathcal{I})}{P(\mathcal{D}|\mathcal{I})}$$

$\mathcal{H}_\alpha \equiv$ Hypothesis $\quad \mathcal{D} \equiv$ Data $\quad \mathcal{I} \equiv$ Prior Information

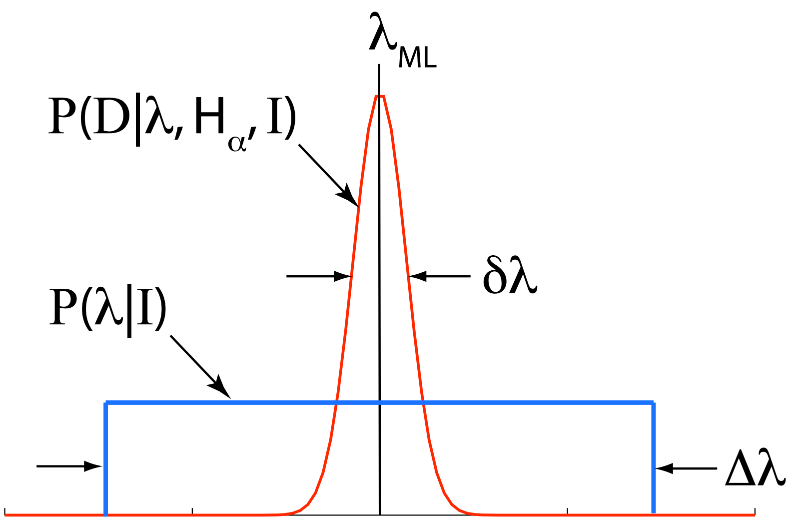
- Odds Ratio:

$$\mathcal{O}_{1,2} = \frac{P(\mathcal{H}_1|\mathcal{D},\mathcal{I})}{P(\mathcal{H}_2|\mathcal{D},\mathcal{I})} = \frac{P(\mathcal{H}_1|\mathcal{I})P(\mathcal{D}|\mathcal{H}_1,\mathcal{I})}{P(\mathcal{H}_2|\mathcal{I})P(\mathcal{D}|\mathcal{H}_2,\mathcal{I})}$$
$$= \frac{P(\mathcal{D}|\mathcal{H}_1,\mathcal{I})}{P(\mathcal{D}|\mathcal{H}_2,\mathcal{I})}$$

Model Evidence and Occam's Factor

- Model Evidence (Global Likelihood for $\mathcal{H}_\alpha$)

$$P(\mathcal{D}|\mathcal{H}_\alpha,\mathcal{I}) = \int P(\vec{\lambda}_\alpha|\mathcal{H}_\alpha,\mathcal{I})P(\mathcal{D}|\vec{\lambda}_\alpha,\mathcal{H}_\alpha,\mathcal{I})\, d\vec{\lambda}_\alpha$$



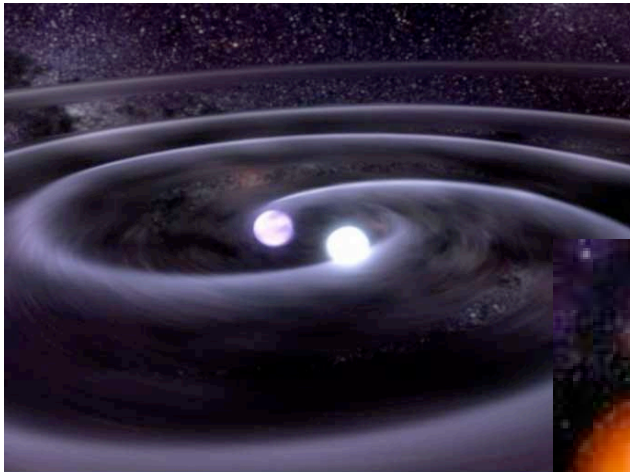
Occam's Factor

$$P(\mathcal{D}|\mathcal{H}_\alpha,\mathcal{I}) \approx P(\mathcal{D}|\lambda_{ML},\mathcal{H}_\alpha,\mathcal{I})\frac{\delta\lambda}{\Delta\lambda}$$

Points of Emphasis

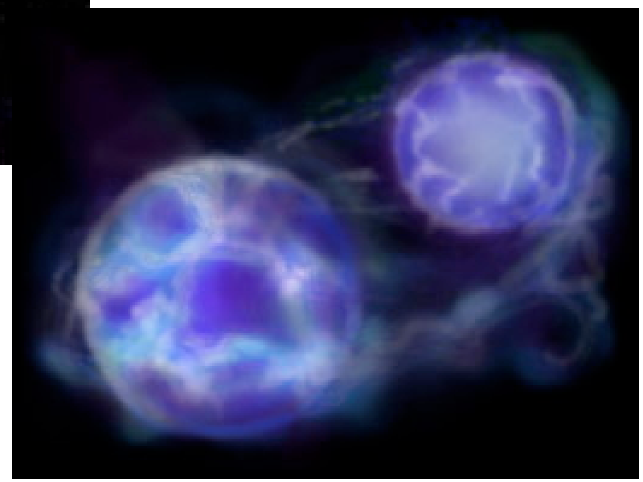
- LISA's data is inherently noisy.
- Parameter estimation is not enough. Models must also be penalized for using too many parameters.

Classes of White Dwarf Binaries



Detached

Semi-detached



Coalescing

$$\frac{df}{dt} = \text{GR} + \text{Astrophysics}$$

Stroeer, Vecchio, & Nelemans, ApJ 633, L33 (2005)

Binary Frequency Evolution

- Given the data and prior information, when are we justified in fitting for a frequency evolution?

Monochromatic model: $\vec{\lambda}_m = \{A, f, \varphi_0\}$

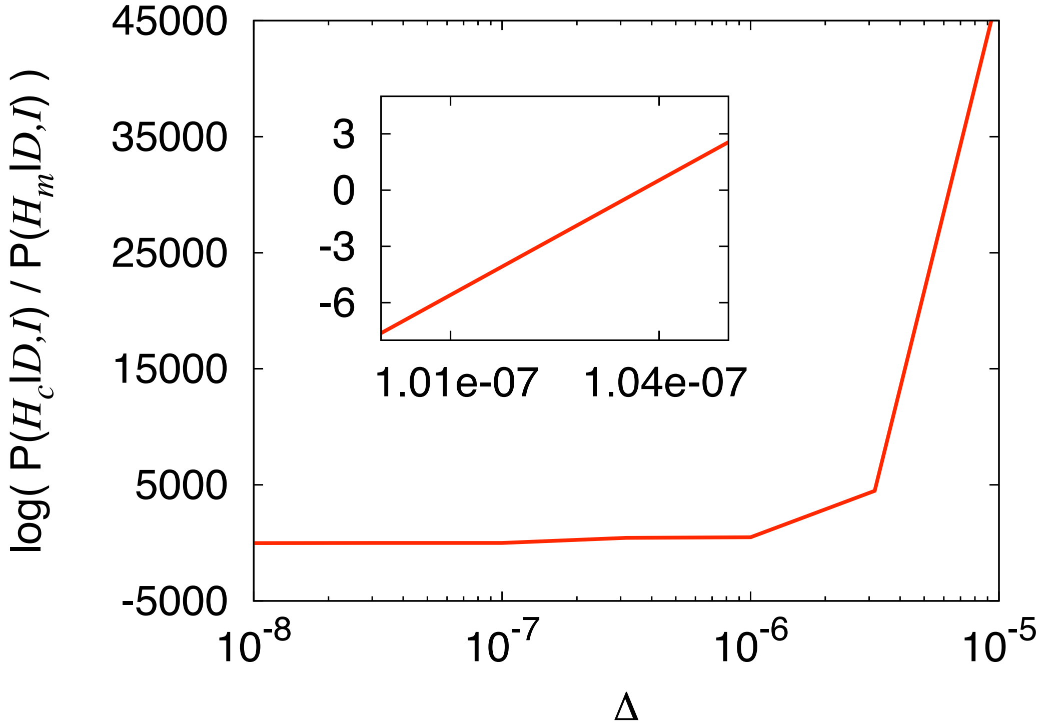
$$\mathcal{H}_m(t; \vec{\lambda}_m) = A\cos(2\pi f t + \varphi_0)$$

Chirping model: $\vec{\lambda}_c = \{A, f, \dot{f}, \varphi_0\}$

$$\mathcal{H}_c(t; \vec{\lambda}_c) = A\cos(2\pi f t + \pi \dot{f} t^2 + \varphi_0)$$

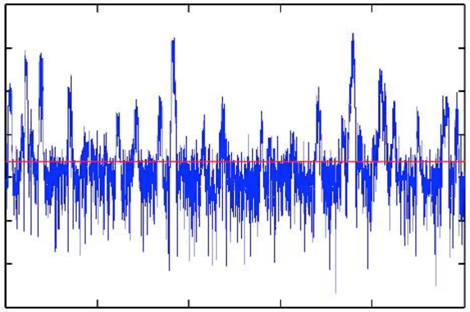
- Given the data and prior information, which model is most probable?

Odds Ratio for the Binary Models

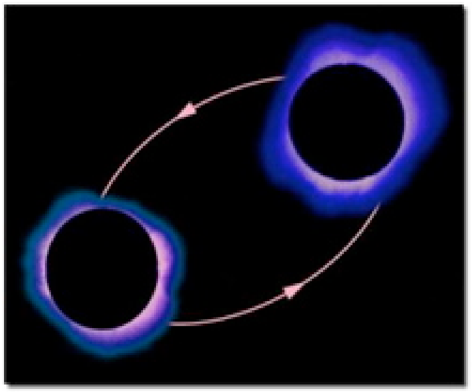


$$\Delta \equiv \frac{\dot{f}T}{f} \qquad N_c = fT = 10^3 \qquad \rho = \frac{A}{\sqrt{2}\sigma} = 20$$

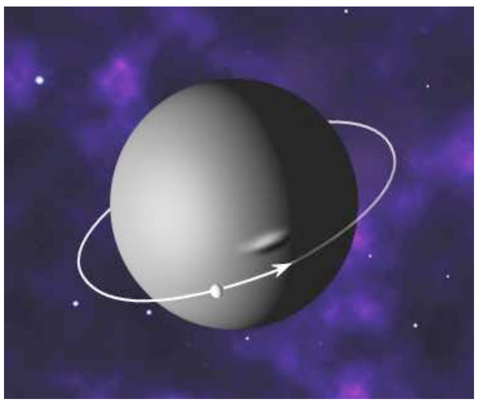
Other LISA Data Applications



How many galactic binary signals are present in this spectrum snippet?



What post-Newtonian order is needed for characterizing a SMBH binary inspiral?



Do we really need to fit for all those EMRI parameters?

Wrap Up

- Before characterizing a signal, we are required to pick a model.
- *Bayesian model comparison gives a logical and quantitative way to directly compare competing models*.
- Bayesian model comparison has a number of applications for LISA data analysis.
- When it comes to the problems of signal detection and characterization we don’t need to use a sledgehammer on a push pin.